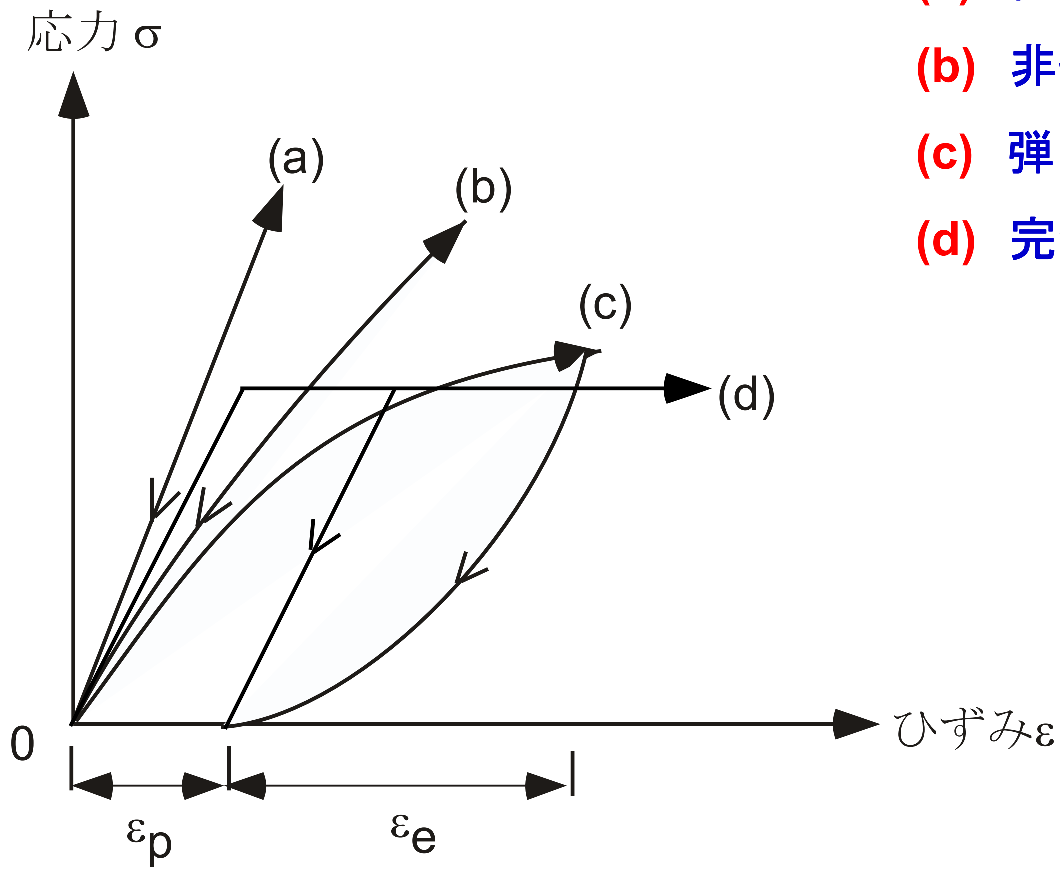
FEM勉強会（第6回，その1）

コンクリートに対する非線形弾性モデルの適用

（前回の復習を含めての解説）

前回の復習：材料非線形問題とは？



(a)　線形弾性

(b)　非線形弾性

(c)　弾・塑性

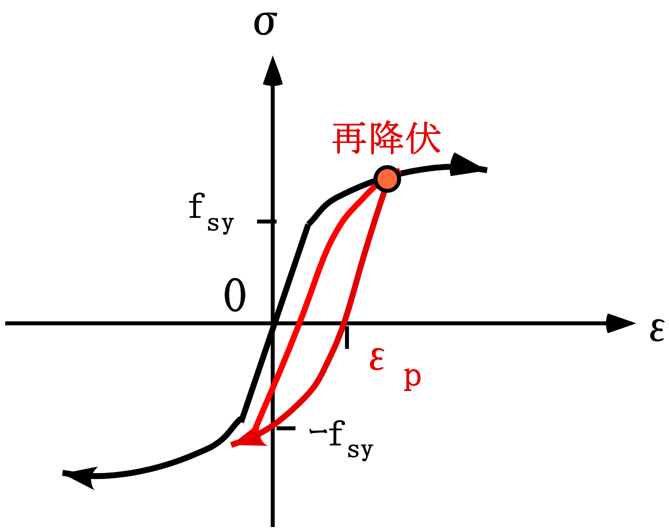
(d)　完全塑性

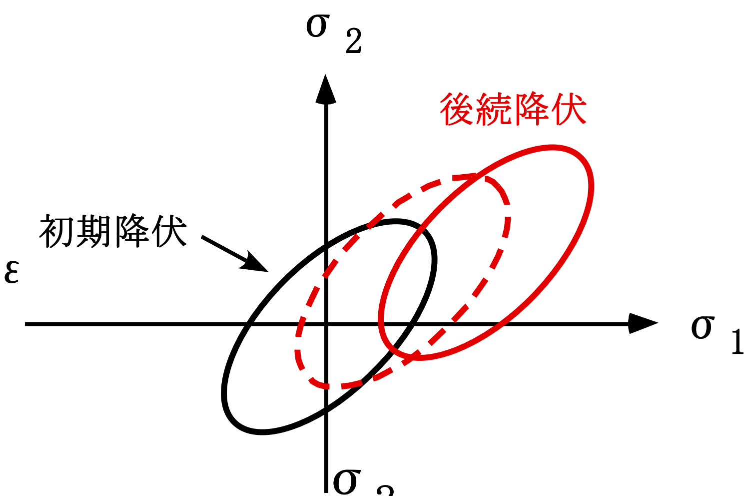
塑性問題：降伏関数、負荷関数、関連流動則

前回の復習

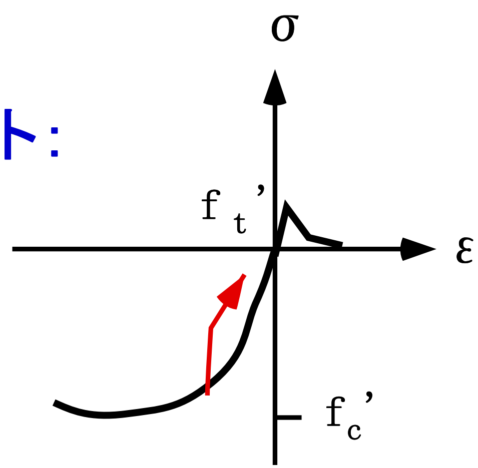
モデルの選択：非線形弾性モデルか、弾塑性モデルか？

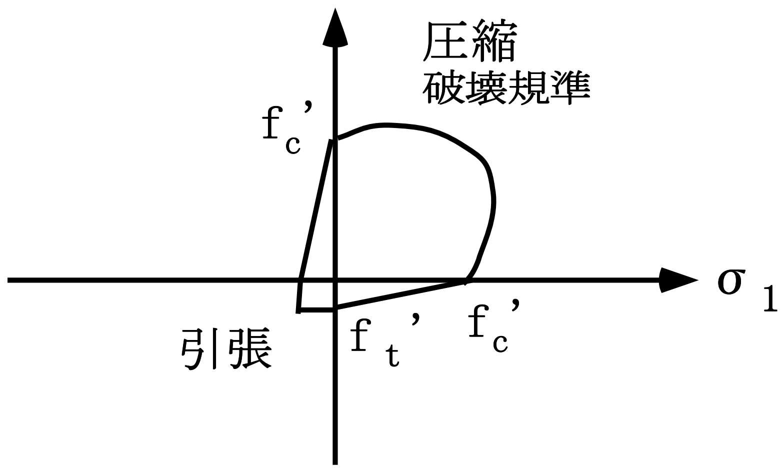
鋼：





コンクリート：



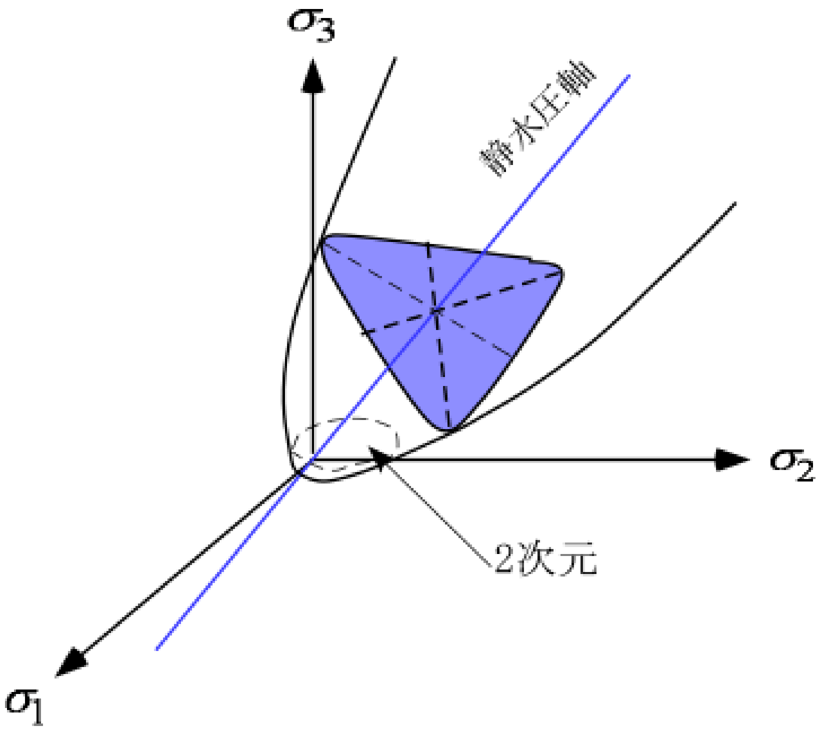


前回の復習:圧縮応力域のコンクリートの取り扱い

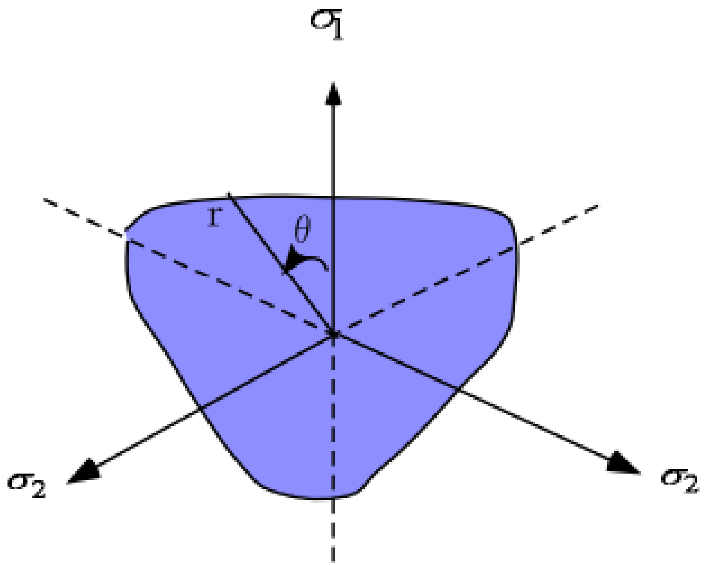
(なぜ八面体応力とひずみが重要なのか？)

$$\sigma_{oct} = \sigma_{kk}/3 \qquad \tau_{oct} = \left(\frac{2}{3}J_2\right)^{1/2} \qquad J_2 = \frac{1}{2}s_{ij}s_{ij} \qquad \sigma_{oct} = 3K_s(\varepsilon_{oct})\cdot\varepsilon_{oct}$$

$$\varepsilon_{oct} = \varepsilon_{kk}/3 \qquad \gamma_{oct} = \left(\frac{8}{3}J_2^e\right)^{1/2} \qquad J_2^e = \frac{1}{2}e_{ij}e_{ij} \qquad \tau_{oct} = G_s(\gamma_{oct})\cdot\gamma_{oct}$$



コンクリートの破壊規準



π 平面上の破壊規準

前回の復習：圧縮応力域のコンクリートの取り扱い

(八面体直応力・ひずみとせん断応力・ひずみの導入)

例題：一軸応力・ひずみ（σ_x-ε_x）関係の多軸応力・ひずみ関係への変換

$$\sigma_{oct} = \frac{\sigma_x}{3} = \frac{E_{c0}}{3} \cdot \varepsilon_x (1 + \bar{a}\varepsilon_x + \bar{b}\varepsilon_x{}^2)$$

$$= \frac{E_{c0}}{1-2\nu_s} \cdot \varepsilon_{oct} \left[1 + \bar{a} \frac{3}{1-2\nu_s} \cdot \varepsilon_{oct} + \bar{b} \left(\frac{3}{1-2\nu_s} \right)^2 \varepsilon_{oct}{}^2 \right]$$

$$\tau_{oct} = \frac{\sqrt{2}}{3}\sigma_x = \frac{E_{c0}}{2(1+\nu_s)} \cdot \gamma_{oct} \left[1 + \bar{a} \frac{3}{2\sqrt{2}(1+\nu_s)} \cdot \gamma_{oct} + \bar{b} \left(\frac{3}{2\sqrt{2}(1+\nu_s)} \right) \cdot \gamma_{oct}{}^2 \right]$$

$$3K_s(\varepsilon_{oct}) = \frac{E_{c0}}{1-2\nu_s} \cdot \left[1 + \bar{a} \frac{3}{1-2\nu_s} \cdot \varepsilon_{oct} + \bar{b} \left(\frac{3}{1-2\nu_s} \right)^2 \varepsilon_{oct}{}^2 \right]$$

$$G_s(\gamma_{oct}) = \frac{E_{c0}}{2(1+\nu_s)} \left[1 + \bar{a} \frac{3}{2\sqrt{2}(1+\nu_s)} \cdot \gamma_{oct} + \bar{b} \left(\frac{3}{2\sqrt{2}(1+\nu_s)} \right) \cdot \gamma_{oct}{}^2 \right]$$

$$\sigma_{ij} = 2G_s(\gamma_{oct})\varepsilon_{ij} + [3K_s(\varepsilon_{oct}) - 2G_s(\gamma_{oct})] \cdot \frac{\varepsilon_{kk}}{3}\delta_{ij}$$

前回の復習：非線形弾性モデルでのFEM解析

時間：t＝0、Δt、2Δt、3Δt、……

t 時刻のひずみと応力

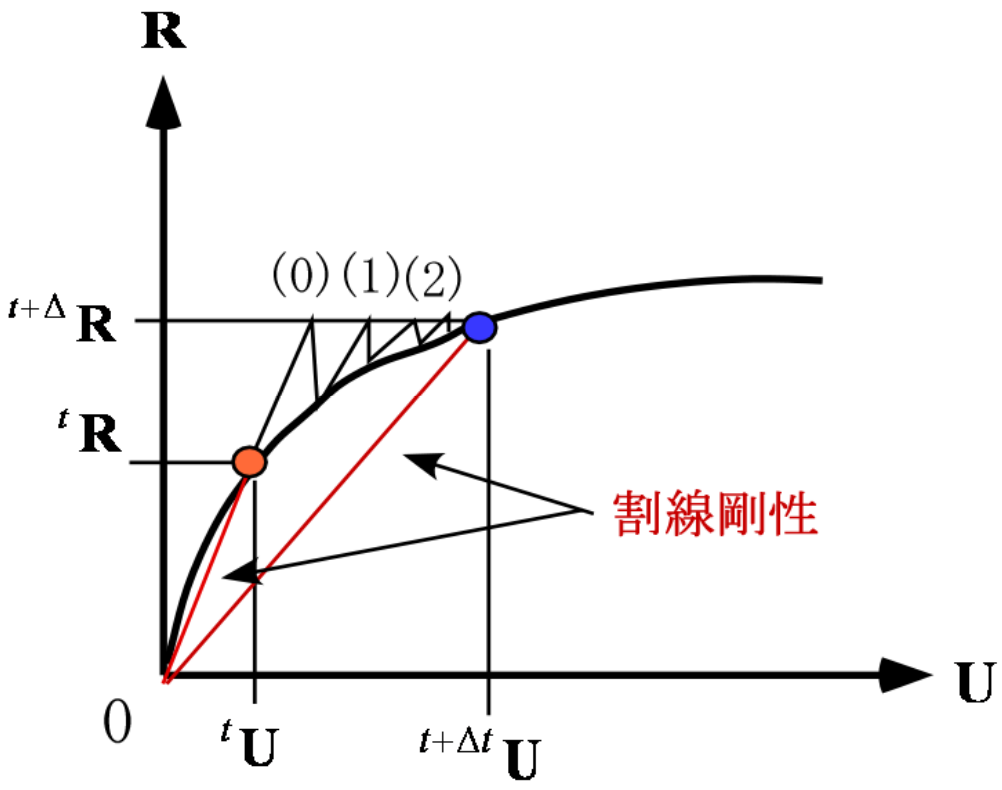
$$ {}^{t}\boldsymbol{\varepsilon}^{(a)} = {}^{t}\mathbf{B}^{(a)}\, {}^{t}\mathbf{U} $$

$$ {}^{t}\boldsymbol{\sigma}^{(a)} = {}^{t}\mathbf{C} \cdot {}^{t}\boldsymbol{\varepsilon}^{(a)} $$

剛性方程式（外力と内力のつりあい）

$$ {}^{t}\mathbf{R} = {}^{t}\mathbf{F} $$

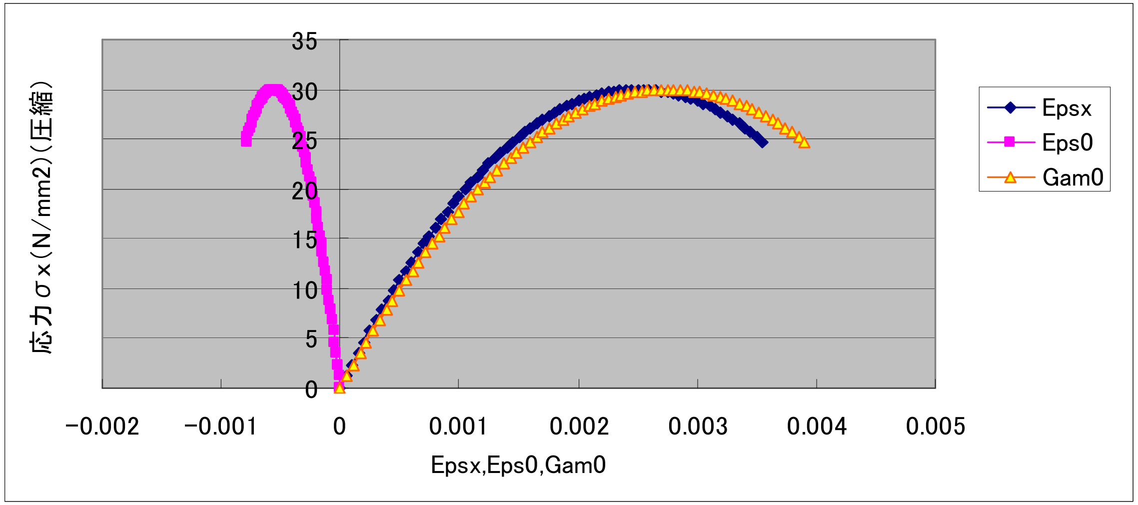
$$ {}^{t}\mathbf{R} = {}^{t}\mathbf{K}\, {}^{t}\mathbf{U} $$

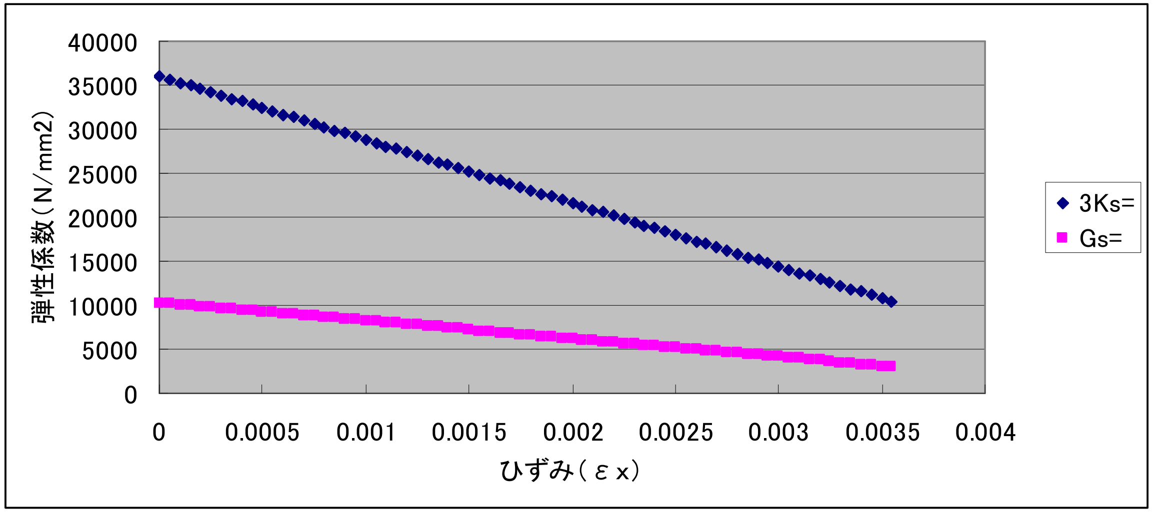


$$ {}^{t}\mathbf{F} = \sum_{(a)} \int_{V^{(a)}} {}^{t}\mathbf{B}^{(a)T}\, {}^{t}\mathbf{C}\, {}^{t}\mathbf{B}^{(a)T} dV^{(a)} \cdot {}^{t}\mathbf{U} = \sum_{(a)} \int_{V^{(a)}} {}^{t}\mathbf{B}^{(a)T}\, {}^{t}\boldsymbol{\sigma} dV^{(a)} $$

$$ {}^{t}\mathbf{K} = \sum_{(a)} \int_{V^{(a)}} \mathbf{B}^{(a)T}\, {}^{t}\mathbf{C}\mathbf{B} dV^{(a)} $$

3次曲線近似: $\sigma = E_{c0}\varepsilon(1+\overline{a}\varepsilon+\overline{b}\varepsilon^2)$ による割線弾性係数の計算結果)





$\nu_s = 1/6$ (一定)

第6回：引張域でのコンクリートの応力・ひずみ曲線の取り扱い

土木学会，コンクリート標準示方書による

3. 2. 4　引張軟化特性

（1）　コンクリートの破壊エネルギー G_F は，一般の普通コンクリートに対して，式（3.2.8）により求めてよい．

$$G_F = 10(d_{max})^{1/3} \cdot f'^{1/3}_{ck} \quad (\mathrm{N/m}) \tag{3.2.8}$$

ここに，d_{max}：粗骨材の最大寸法　（mm）

f'_{ck}：圧縮強度の特性値（設計基準強度）（N/mm²）

（2）　引張軟化曲線は，図 3. 2. 2 に示したモデル化されたものを使用してもよい．

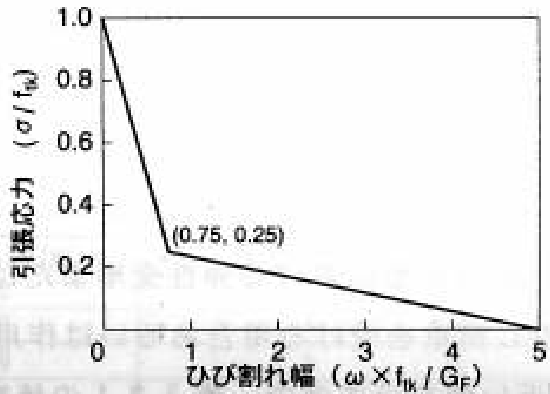


図 3. 2. 2 引張軟化曲線

G_F=60-130N/m

$f_{tk}=0.23f'^{\,2/3}_{ck}$

割線弾性係数法による反復・収束計算

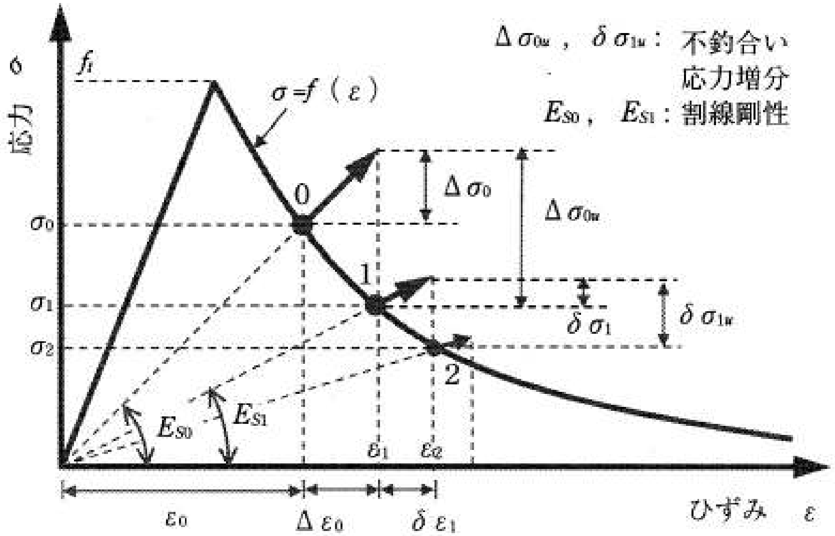


図 5.30　反復計算による引張軟化曲線の追跡

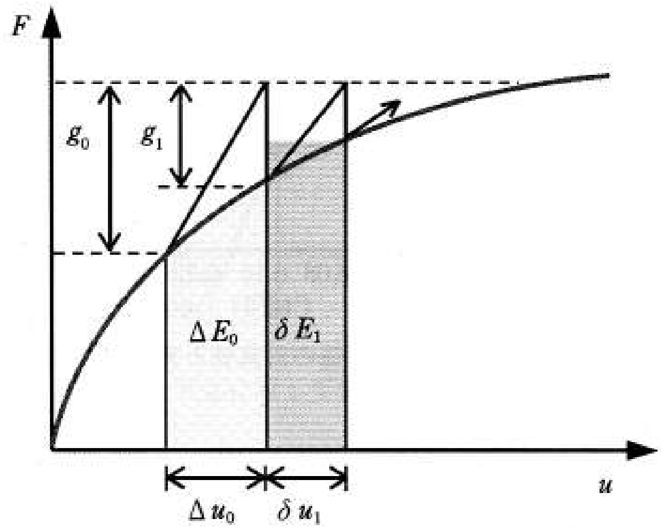


図 5.31　Newton-Raphson 法による反復過程

Bazant（1983）:

$\varepsilon = \omega / h$

h：ひび割れ帯幅

参考文献:

三橋，六郷ら：コンクリートのひび割れと破壊の力学，技報堂出版(2011)

ひび割れ帯幅hの導入（Bazant(1983)），h＝λ，ε＝ω／λ

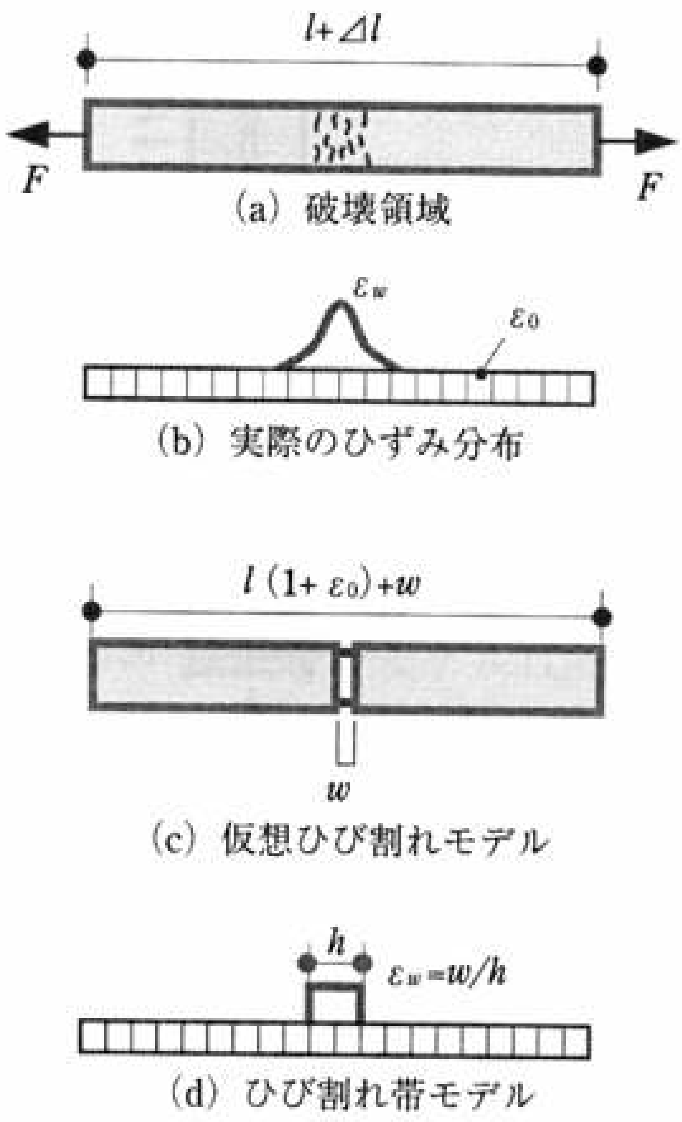


図 5.20 引張破壊のモデル化

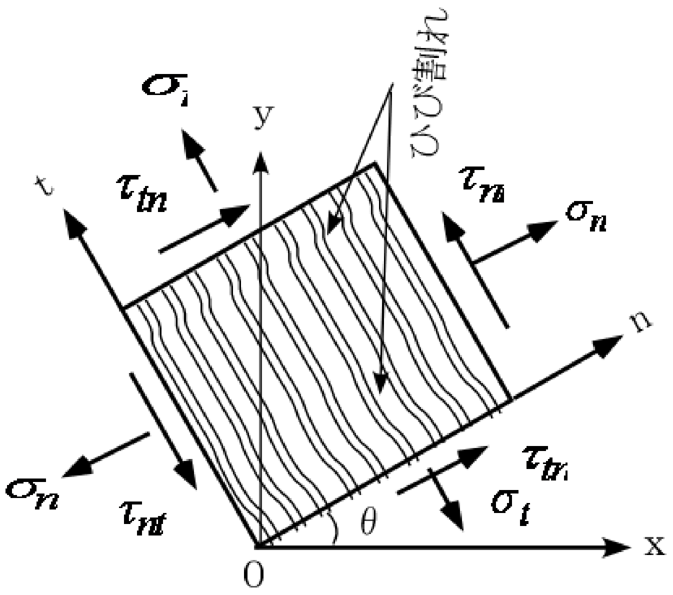
ひび割れ分散モデルによるFEM解析での定式化

ひび割れ要素での構成則：　$\boldsymbol{\sigma}^{cr} = \mathbf{E}^{cr}\boldsymbol{\varepsilon}^{cr}$　　$\boldsymbol{\sigma}^{cr} = [\sigma_n \quad \sigma_t \quad \tau_{nt}]$

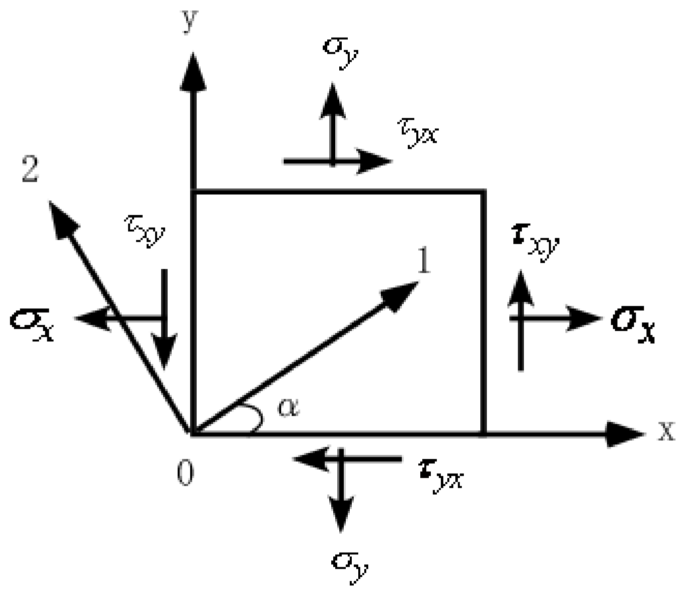
$$\mathbf{E}^{cr} = \begin{bmatrix} E_n^{'} & 0 & 0 \\ 0 & E_c & 0 \\ 0 & 0 & \beta G_c \end{bmatrix}$$

$\boldsymbol{\varepsilon}^{cr} = [\varepsilon_n \quad \varepsilon_t \quad \gamma_{nt}]$

β　は係数



ひび割れ要素（平面要素）



主応力の方向

主応力とその方向

$$E_{\mathrm{ncr}} = \alpha \tan \theta_t$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} \qquad \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

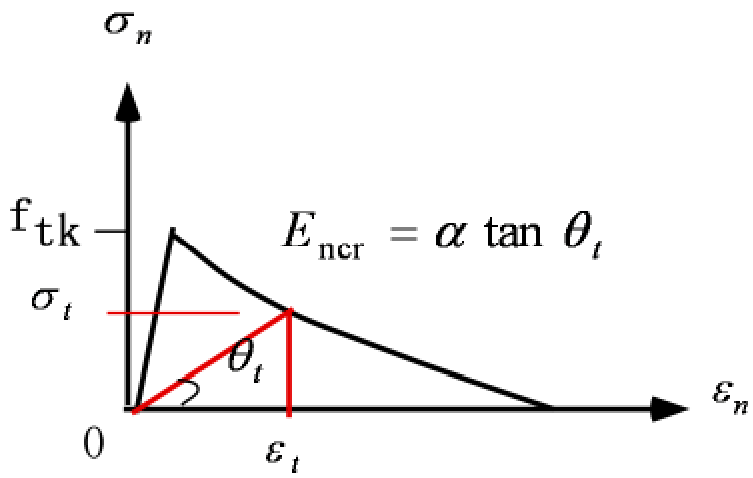
$$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \alpha = \frac{1}{2}\arctan\left|\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right|$$

ただし, $0 \le \alpha \le \frac{\pi}{4}$

$$\varepsilon_n = \varepsilon_{cr} + \omega / \lambda \qquad \boldsymbol{\sigma}^{cr} = \mathbf{E}^{cr}\boldsymbol{\varepsilon}^{cr}$$

$$\mathbf{E}^{cr} = \begin{bmatrix} \alpha E_c & \alpha \nu_c E_c & 0 \\ \alpha \nu_c E_c & E_c & 0 \\ 0 & 0 & \beta G_c \end{bmatrix}$$

$$\alpha = \sigma_t / f_{tk} \qquad 0 \le \alpha \le 1$$



座標変換

$$\boldsymbol{\sigma} = \mathbf{T}_\sigma \boldsymbol{\sigma}^{cr} \qquad \mathbf{T}_\sigma = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \sin 2\theta \\ \sin^2\theta & \cos^2\theta & -\sin 2\theta \\ -\dfrac{\sin 2\theta}{2} & \dfrac{\sin 2\theta}{2} & \cos 2\theta \end{bmatrix}$$

$$\boldsymbol{\varepsilon} = \mathbf{T}_\varepsilon \boldsymbol{\varepsilon}^{cr} \qquad \mathbf{T}_\varepsilon = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \dfrac{\sin 2\theta}{2} \\ \sin^2\theta & \cos^2\theta & -\dfrac{\sin 2\theta}{2} \\ -\sin 2\theta & \sin 2\theta & \cos 2\theta \end{bmatrix}$$

θ : 主引張方向

$\boldsymbol{\sigma}$ $\boldsymbol{\varepsilon}$: x,y軸に関する応力とひずみベクトル

$$\boldsymbol{\sigma} = \widetilde{\mathbf{E}}\boldsymbol{\varepsilon} \qquad \widetilde{\mathbf{E}} = \mathbf{T}_\sigma^{-1}\mathbf{E}^{cr}\mathbf{T}_\varepsilon$$

$$\mathbf{K}^{cr} = \int_{V^{(m)}} \mathbf{B}^{(m)}\widetilde{\mathbf{E}} \cdot \mathbf{B}^{(m)^T} dV^{(m)}$$

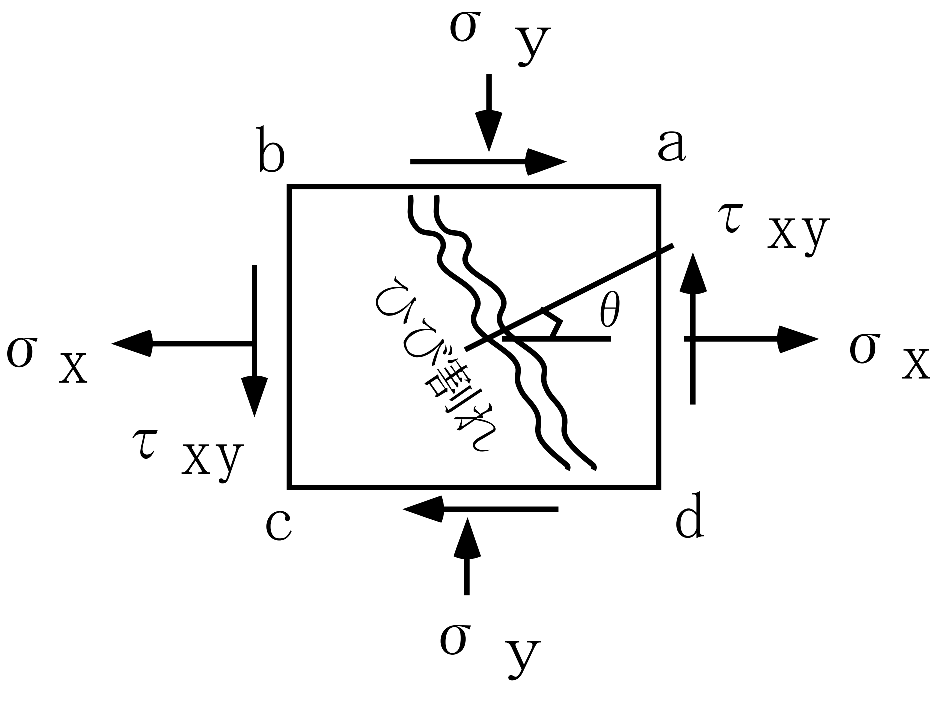
要素テスト：数値計算による検討

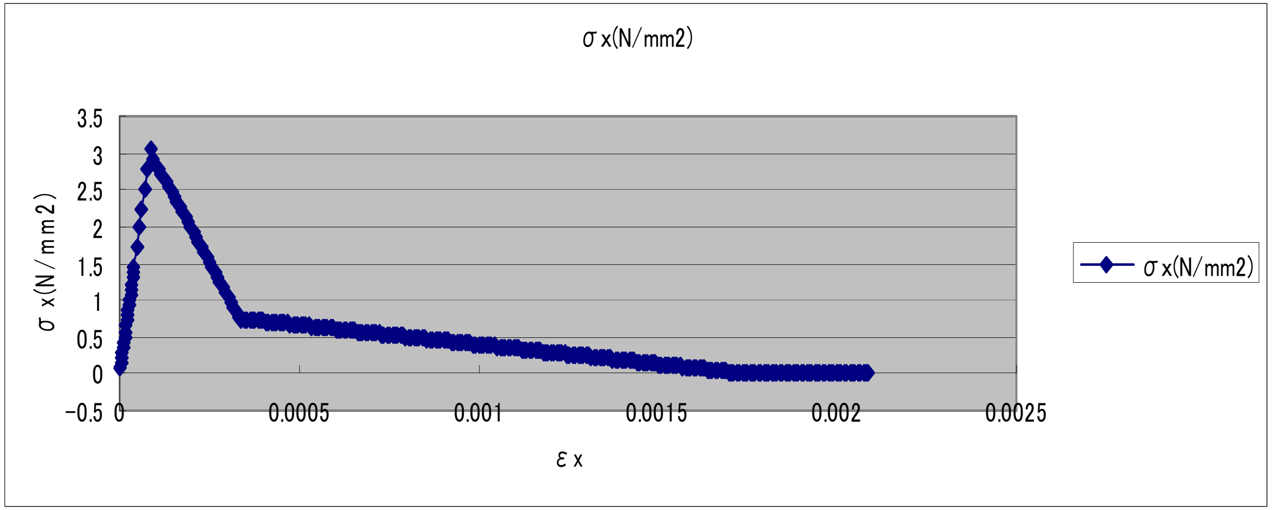
平面応力要素a-b-c-dを取り上げ，x方向に引張ひずみ(ε_x)，y方向に圧縮ひずみ(-ε_y)，そしてせん断ひずみ(γ_{xy})を受けた時の応力(σ_x, σ_y, τ_{xy})の応答を求める.

入力条件：

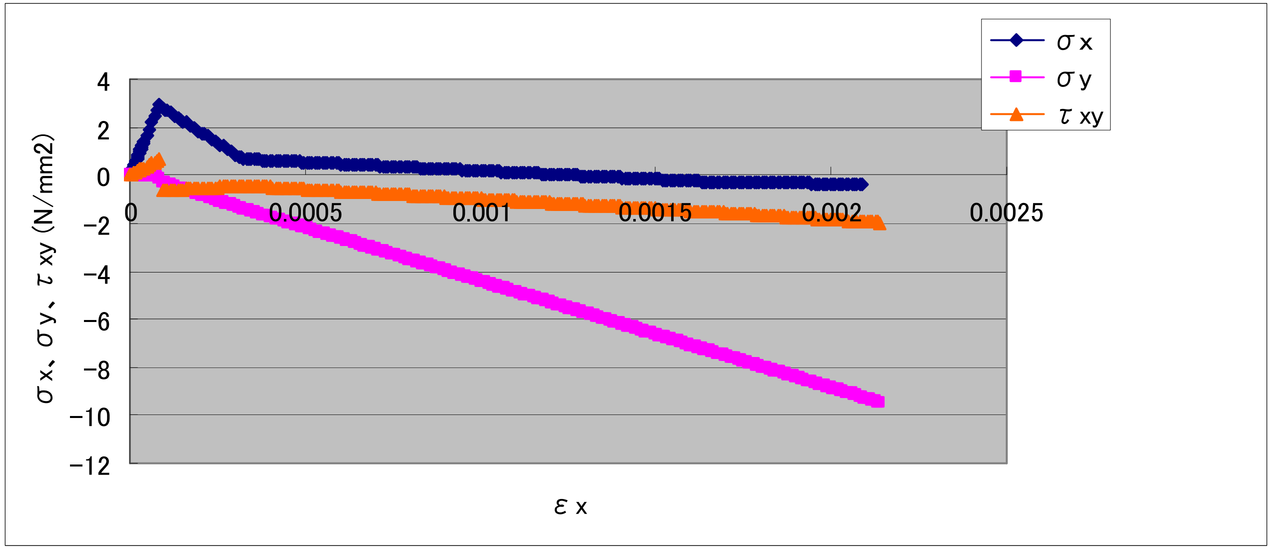
Ec=3x10⁴N/mm², $\nu_c = 1/6$ GF=100N/m, $f_t' = 3\mathrm{N/mm^2}$

$\beta = G_{cr}/G \quad \varepsilon_y = -\beta_{sy}\varepsilon_x \quad \gamma_{xy} = \beta_t \varepsilon_x \quad \lambda = 100\mathrm{mm}$





ケース1：β =0.5，β_{sy}=β_t＝0



ケース2：β =0.5，β_{sy}=0.2，β_t＝0.5

参考　エキセル・マクロ・VBAのプログラミング

```
Private Sub CommandButton1_Click()

    Dim EM0(3, 3), EMCR(3, 3), Tsigm(10, 10), Teps(3, 3), AMW(3, 3), BMW(3, 3)
    Dim AA(100, 100), Aeps(3), Deps(3), ASigm(3)

    N = 3

    Ec = 30000#: Ft = 3#: Rniu = 1# / 6#
    Gc = Ec / (2# * (1# + Rniu))
    Epstx0 = Ft / Ec


    Sheet1.Cells(1, 1) = "Ft(N/mm2)=": Sheet1.Cells(1, 2) = Ft: Sheet1.Cells(1, 3) =
"Ec(N/mm2)=": Sheet1.Cells(1, 4) = Ec
    Sheet1.Cells(1, 5) = "Rniu=": Sheet1.Cells(1, 6) = Rniu
```

```
'Input Data
    BetaSY = 0.2: BetaT = 0.5
    GF = 0.1: Omeg = GF / Ft
    Ramda = 100#: EpsCrack = Omeg / Ramda

    Bcrack = 0.5

    Pai = 3.141569

    W = Ec / (1# - Rniu)
    EM0(1, 1) = 1# * W: EM0(1, 2) = Rniu * W: EM0(1, 3) = 0#
    EM0(2, 1) = Rniu * W: EM0(2, 2) = 1# * W: EM0(2, 3) = 0#
    EM0(3, 1) = 0#: EM0(3, 2) = 0#: EM0(3, 3) = W * (1# - Rniu) / 2#


    Epsmax = 0#: IP = 0: Alp = 0#: Epscr0 = 0#
```

```
For i = 1 To 300

   If i <= 20 Then Epsx = 0.02 * i * Epstx0
   If i > 20 Then Epsx = (0.4 + (i - 20) * 0.075) * Epstx0


   Epsy = -BetaSY * Epsx
   Gamxy = BetaT * Epsx

   Aeps(1) = Epsx: Aeps(2) = Epsy: Aeps(3) = Gamxy

   For k = 1 To 3
   W = 0#
   For L = 1 To 3
   W = W + EM0(k, L) * Aeps(L)
   Next L
   ASigm(k) = W
   Next k
```

```
Sigmx = ASigm(1): Sigmy = ASigm(2): Tauxy = ASigm(3)

    W = 0.25 * (Sigmx - Sigmy) ^ 2 + Tauxy ^ 2
    Sigm1 = 0.5 * (Sigmx + Sigmy) + Sqr(W)
    Sigm2 = 0.5 * (Sigmx + Sigmy) - Sqr(W)
    W = 0.25 * (Epsx - Epsy) ^ 2 + 0.25 * Gamxy ^ 2
    Eps1 = 0.5 * (Epsx + Epsy) + Sqr(W)
    Eps2 = 0.5 * (Epsx + Epsy) - Sqr(W)


    If Sigm1 <= Ft And IP = 0 Then
    If Aeps(1) > Epsmax Then Epsmax = Aeps(1)

 Sheet1.Cells(i + 2, 1) = Aeps(1): Sheet1.Cells(i + 2, 2) = Sigmx
   Sheet1.Cells(i + 2, 4) = Aeps(2): Sheet1.Cells(i + 2, 5) = Sigmy
   Sheet1.Cells(i + 2, 7) = Aeps(3): Sheet1.Cells(i + 2, 8) = Tauxy
   Sheet2.Cells(i + 2, 1) = Eps1: Sheet2.Cells(i + 2, 2) = Depscr / EpsCrack:
Sheet2.Cells(i + 2, 3) = FtCrack / Ft
   Sheet2.Cells(i + 2, 4) = Theta * 57.3
   IP = 0
```

```
 ElseIf Sigm1 > Ft And IP = 0 Then

   W = 2# * Tauxy / (Sigmx - Sigmy)
   ALp0 = 0.5 * Atn(W)
   IP = i: Epscr0 = Eps1

      If Sigmx > Sigmy And Tauxy > 0# Then
      Alp = ALp0
      ElseIf Sigmx > Sigmy And Tauxy < 0# Then
      Alp = Pai - ALp0
      ElseIf Sigmx < Sigmy And Tauxy > 0# Then
      Alp = 0.5 * Pai - ALp0
      ElseIf Sigmx < Sigmy And Tauxy < 0# Then
      Alp = 0.5 + ALp0
      Else
      End If

      Theta = Alp

      CC2 = Cos(Theta) ^ 2: SS2 = Sin(Theta) ^ 2: S2T = Sin(2# * Theta): C2T =
CC2 - SS2
```

```
Tsigm(1, 1) = CC2: Tsigm(1, 2) = SS2: Tsigm(1, 3) = S2T
      Tsigm(2, 1) = SS2: Tsigm(2, 2) = CC2: Tsigm(2, 3) = -S2T
      Tsigm(3, 1) = -0.5 * S2T: Tsigm(3, 2) = 0.5 * S2T: Tsigm(3, 3) = C2T

      Teps(1, 1) = CC2: Teps(1, 2) = SS2: Teps(1, 3) = 0.5 * S2T
      Teps(2, 1) = SS2: Teps(2, 2) = CC2: Teps(3, 3) = -0.5 * S2T
      Teps(3, 1) = -S2T: Teps(3, 2) = S2T: Teps(3, 3) = C2T


      Matinv Tsigm(), N

      Sheet1.Cells(i + 2, 1) = Aeps(1): Sheet1.Cells(i + 2, 2) = Sigmx
      Sheet1.Cells(i + 2, 4) = Aeps(2): Sheet1.Cells(i + 2, 5) = Sigmy
      Sheet1.Cells(i + 2, 7) = Aeps(3): Sheet1.Cells(i + 2, 8) = Tauxy
      Sheet2.Cells(i + 2, 1) = Eps1: Sheet2.Cells(i + 2, 2) = Depscr /
EpsCrack: Sheet2.Cells(i + 2, 3) = FtCrack / Ft
```

```
Else
      Depscr = Eps1 - Epscr0
      W = Ft

      If Depscr <= 0.75 * EpsCrack Then W = Ft * (1# - Depscr / EpsCrack)
      If Depscr > 0.75 * EpsCrack And Depscr < 5# * EpsCrack Then W =
0.294 * Ft * (1# - 0.2 * Depscr / EpsCrack)
      If Depscr > 5# * EpsCrack Then W = 0#

      FtCrack = W
      EcCrack = FtCrack / Eps1

    EMCR(1, 1) = EcCrack: EMCR(1, 2) = 0#: EMCR(1, 3) = 0#
         EMCR(2, 1) = 0#: EMCR(2, 2) = Ec: EMCR(2, 3) = 0#
         EMCR(3, 1) = 0#: EMCR(3, 2) = 0#: EMCR(3, 3) = Bcrack * EM0(3, 3)

         For k = 1 To 3
         For L = 1 To 3
         W = 0#
         For M = 1 To 3
         W = W + EMCR(k, M) * Teps(M, L)
         Next M
```

```
AMW(k, L) = W
    Next L
    Next k

    For k = 1 To 3
    For L = 1 To 3
    W = 0#
    For M = 1 To 3
    W = W + Tsigm(k, M) * AMW(M, L)
    Next M
    BMW(k, L) = W
    Next L
    Next k

    W = 0#
    For k = 1 To 3
    W = 0#
    For L = 1 To 3
    W = W + BMW(k, L) * Aeps(L)
    Next L
```

```
ASigm(k) = W
      Next k

      Sheet1.Cells(i + 2, 1) = Aeps(1): Sheet1.Cells(i + 2, 2) = ASigm(1)
      Sheet1.Cells(i + 2, 4) = Aeps(2): Sheet1.Cells(i + 2, 5) = ASigm(2)
      Sheet1.Cells(i + 2, 7) = Aeps(3): Sheet1.Cells(i + 2, 8) = ASigm(3)
      Sheet2.Cells(i + 2, 1) = Eps1: Sheet2.Cells(i + 2, 2) = Depscr / EpsCrack:
Sheet2.Cells(i + 2, 3) = FtCrack / Ft
      Sheet2.Cells(i + 2, 4) = Theta * 57.3


  End If

   Next i

     Stop

  End Sub
```